CONSTRUCTION AND ANALYSIS OF CONFOUNDED ASYMMETRICAL FACTORIAL DESIGNS WITH SOME FACTORS HAVING LEVELS EQUAL TO PRIME POWER

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INTRODUCTION

In asymmetrical factorial experiments, if the levels of each factor is a prime number, then there exists a number of methods of constructing the designs and analysing the data therefrom given by Das (1960), Kishen and Srivastava (1959) and several others. But if the levels of atleast one of the factors is not a prime number, but prime power then there is no suitable method for constructing such designs and for analysis. An attempt has been made in this paper to solve this problem.

Here a method has been discussed for constructing designs of the type $p^m \times q^n$ with blocks of size $p^t \times q^n$, where m and n are two positive integers indicating number of factors, p is a prime power and q is a prime number, indicating levels of each of m factors and that of n factors respectively. Again it is also a positive integer. Method of analysis is also cited together. Though the method has been explained with only two types of factors, it can be easily extended with any number of different factors.

2. Method of Construction

If the number of levels of factors be a prime power, then the corresponding prime number will be one of 2, 3, 5, etc. Now the number of factors to be considered for the single factor with prime power levels must be equal to the power of the prime number. Then the design is to be constructed with the number of factors considered a new following Das's method. From this new design designated as parent design, final design is to be obtained by combining the groups of factors, each group of factors being originated from an individual factor with levels of prime power. Suppose a design of type $p^m \times q^n$ with blocks of size $p^t \times q^n$ is to be constructed. And again suppose $p=g^b$ where g is a prime number and b is a positive integer. The parent design is to be constructed with the factors of size $bm+n$, of which bm factors have levels g each and the rest n factors have levels q each. Here instead of m factors with p levels each, bm factors have been taken for constructing the design following the existing method. Then to make the final design, bm factors, each with g levels in the parent design have to be grouped into m groups, each consisting of b factors such that a combination of b factors in a group will form a level of a factor with p levels. Putting certain codes for all these combinations of m groups each having b factors, the final design with original factors can be brought.

3. Method of Analysis

While constructing the parent design by considering $bm+n$ factors certain interactions are to be confounded completely or partially. All these completely or partially confounded interactions will form a part or parts of an interaction or interactions consisting of the original factors. Now by making contrasts of all these interactions from the parent design and finding out their sums of squares, confounded portion can be removed from the S.S. of the original interaction or interactions of which these confounded interaction or interactions form a part, by subtractions. Then the S.S. of the interactions from the parent design have to be adjusted by being divided by their respective informations gained. These adjusted S.S. are to be added to the corresponding subtracted S.S. of the original interaction or interactions to get the adjusted S.S. due to effected interaction or interactions with the original factors.

For finding out the S.S. due to main effects and interactions a $(m+n)$-way table is to be made, each cell total being adjusted for the blocks as described by Ray and Das (Ray's Diploma Dissertation 1969). From this $(m+n)$-way table, by partitioning, the S.S. due to any main effect or interaction can be obtained. S.S. obtained from this $(m+n)$-way table will give S.S. due to non-confounded main effects or interactions. But for effected interaction some adjustment is necessary and this adjustment is made as described above.

4. Example

Here the method of construction and analysis of this special type of design has been explained by giving an example of design of $4\times4\times3$ in 12 plot blocks.

Here factors are A, B, and X where each of A and B has got 4 levels and X has got 3 levels. 4 being a power of 2, two factors A_1 and A_2 for A and B_1 and B_2 for B have been considered for conducting the parent design each of A_1, A_2, B_1 and B_2 having 2 levels. Now while constructing the parent design the new series has been of the type $2\times2\times2\times2\times3$ in 12 plot blocks. Now in the parent design interactions A_1B_1, A_1B_1X, A_2B_2X are partially confounded and $A_1A_2B_1B_2$ is completely confounded. Here interactions A_1B_1, A_2B_2 and $A_1A_2B_1B_2$ each with 1 *d.f.* form parts of the interaction $A\ B$ with given factors and similarly interactions A_1B_1X and A_2B_2X, each with 2 df form parts of the interaction ABX with given factors. Combining A_1 and A_2 together and B_1 and B_2 together the final design with factors A, B and X is obtained.

5. Summary

Here an attempt has been made to obtain designs for the factorial experiments having factors with unequal levels, some factors having number of levels which is a prime power. Analysis of such designs is also discussed. The method of construction and analysis of such type of design is illustrated with the help of an example.

REFERENCES

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